The Critical Angle and Percent Efficiency of Parabolic Solar Cookers

Ariel Chen

Abstract:

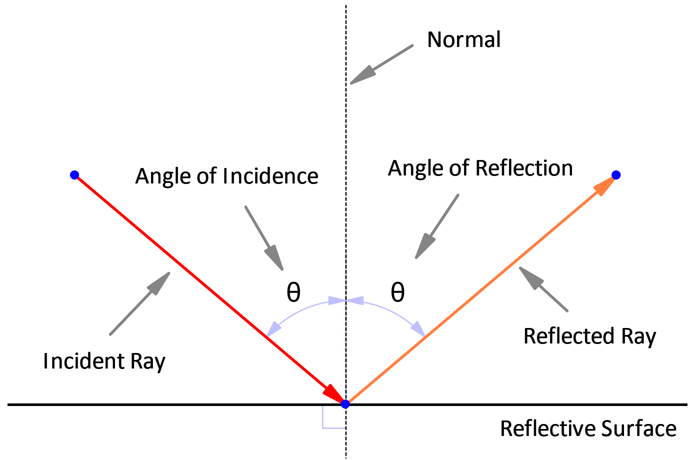
The parabola is commonly used as the curve for solar cookers because of its ability to reflect incoming light with an incoming angle of 90 degrees to a single focus point. However, because of this very specific property, the parabolic solar cooker must constantly be realigned with the orbiting sun for maximum effectiveness. Solar cookers concentrate light to a larger surface, so it is slightly more tolerant for marginal errors of alignment. However, to what point are the reflected rays no longer tolerant, and start to miss the target? How quickly do the rest of the reflected rays miss? Is there an optimal focal height or target size that will always be more tolerant of errors in alignment? Using Geometry Expressions, a constraint-based symbolic geometry system, in conjunction with Maple, a computer algebra system, these situations were modeled and investigated. We found that the solar concentrator always has the largest tolerance for error when the focal height is a fourth of the diameter, or level with the boundaries of the concentrator. The percentage of light reflected to the target is at 100% for a small range of angles, and promptly completely misses the target at angles outside this range. This miss, called the spill rate, is essential for comparing the parabola with other curves. Although there may not be another curve more accurate than the parabola when perfectly aligned with the light source, there may be another curve that is more accurate at wider angles.

http://www.blog.thesietch.org/wp-content/uploads/2007/02/vajra-2006-1.JPG

Introduction:

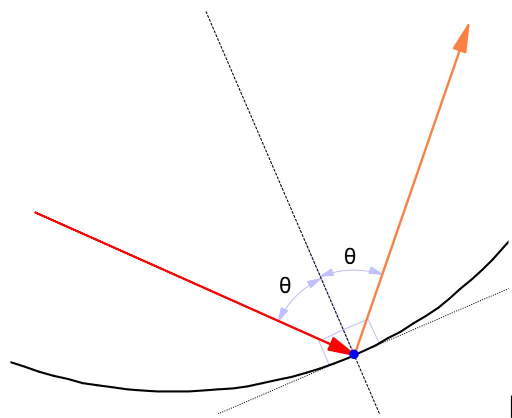
Solar cookers have become popular around the world because they heat and cook without consuming fuels which are not always readily available. They can reach higher temperatures than the surrounding atmosphere because solar cookers concentrate a large amount of light to a relatively smaller absorbing surface. The area that sunlight is collected from is called the aperture area. Before how solar cookers concentrate light is discussed, the law of reflection must first be understood.

The diagram [1] below illustrates how a single ray of light reflects off a flat mirror. The incoming ray, called the incident ray, hits the mirror at the incidence point. At the incident point, a line perpendicular to the mirror, the normal, acts as a line of symmetry for the incident ray and the reflected ray. The angle between the reflected ray and the normal, the angle of reflection, and the angle between the incident ray and the normal, the angle of incidence, are always identical.



[1]

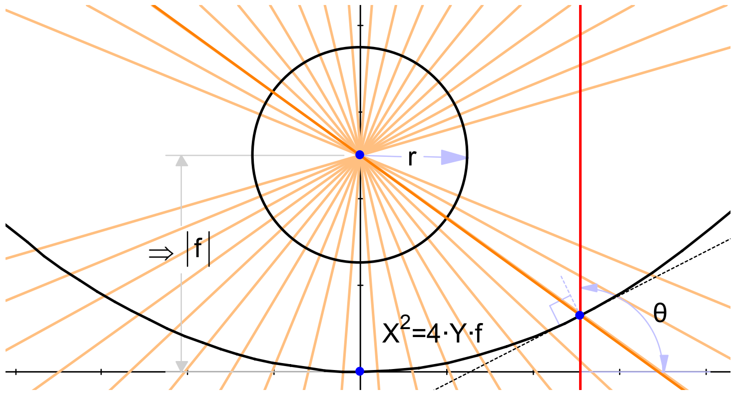
For curved surfaces, the normal is perpendicular to the tangent at the incidence point shown in the diagram [2] below.



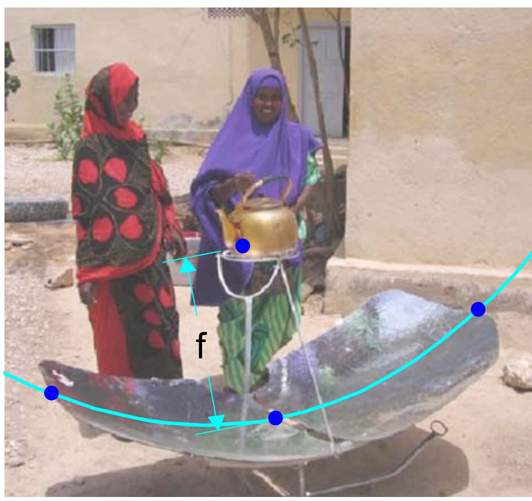
[2]

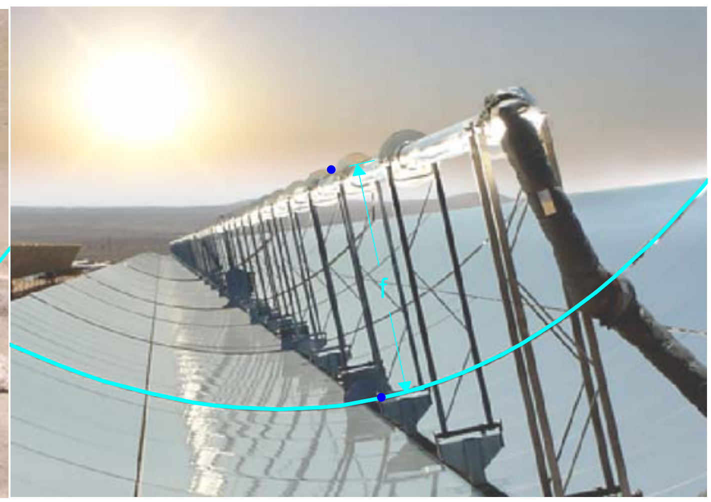
Solar cookers come in all different shapes and sizes, but because of the parabola's ability to reflect light onto a single focus point, parabolic cookers are commonly used. However, this special property is only useful when light has an incoming angle of 90^{o}. The reflected rays completely miss the focal point at any other incoming angle. Because solar cookers are concentrating light to a surface (the pot), rather than a single focus point, the solar cooker can still function at angles slightly less than 90^{o}.

In the model [3] created with Geometry Expressions, we examine the properties of the reflected light rays in a 2D view. The parabola is given the general equation of $x^2 = 4y \times f$, where *f* is the focal height. The red line represents the indefinite number of incoming rays, the orange lines represent all the reflected rays, Θ represents the incoming angle of light, and the circle represents a pot with radius *r.*



This simulates real-world parabolic solar concentrators such as the ones shown below.

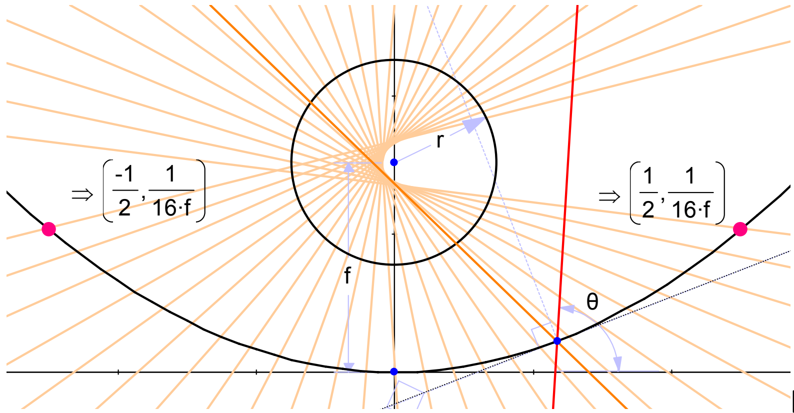




http://digitaljourno.wordpress.com/2008/03/14/sun-fire-and-cooking/ http://www.ecoworld.com/fuels/solar-thermal-power.html

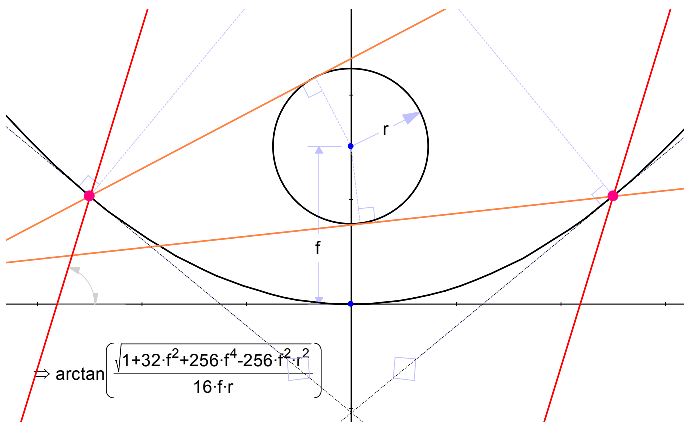
Problem – Finding the Critical Angle:

Because the position of the sun in the sky is constantly changing, we are interested in determining the focal height and radius that will collect the most reflected rays with the most angle tolerance. If a solar cooker has a large angle tolerance, it means that it is able to reflect light onto the absorbing surface at extreme or smaller incoming angles. For a generic model [4] the aperture length of the solar cooker is held constant at 1 while the radius (*r)* and focal height (*f*) are interpreted as ratios of the diameter.



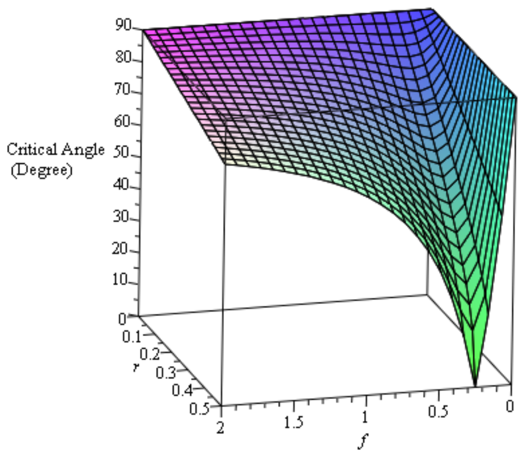
[4]

In this particular situation [4], all the reflected light rays are hitting the pot. But at what angle *Θ* do the light rays start missing the pot? Geometry Expressions calculates the critical incoming angle that light starts missing the pot as $\arctan\left(\frac{1}{16}\,\frac{\sqrt{1+32f^2+256f^4-256r^2f^2}}{rf}\right)$. This critical angle is the widest incoming angle for a given *r* and *f* and an aperture length of 1 that concentrates all the reflected rays to the pot with radius *r*.

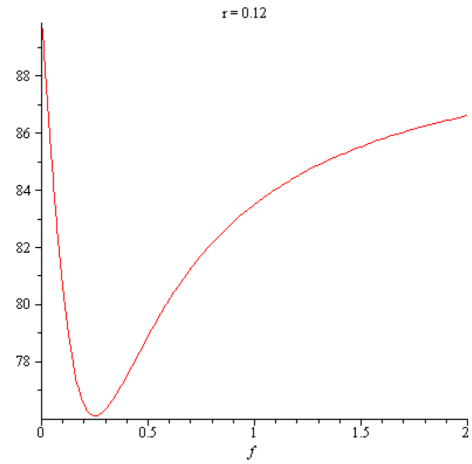


[5]

To further understand the meaning of this formula, the angle formula was graphed using Maple. The graph of the formula and a cross section at where *r*=0.12 is shown:



[6]



[7]

These graphs [6],[7] show the critical angle for a given *f* and *r* where the reflected rays start to miss the pot. Because our goal is to determine the combination that will have the largest angle tolerance, the smaller the critical angle, the more angle tolerant the solar cooker.
As visible from the graph [6], the solar cooker becomes more angle tolerant as the pot size (*r)* increases. This is expected because as *r* increases, the reflected rays have a larger absorbing area to hit. However, having a large radius means that there will be a smaller concentration ratio (aperture area to absorbing area ratio).

From observing the graph, it becomes apparent that there is always a focal height that will have the minimal critical angle. To solve for the minimum, the angle formula $\arctan\left(\frac{1}{16}\frac{\sqrt{1+32f^2+256f^4-256r^2f^2}}{rf}\right)$ was differentiated and solved for in Maple:

```
> arctan((((1+((f)^(2)*32)+((f)^(4)*256)+((r)^(2)*(f)^(2)*(-256))))^(1/2)*(r)^((-1))*(f)^((-
1))*1/16));
```

$$\arctan\left(\frac{\sqrt{1+32\,f^2+256\,f^4-256\,r^2\,f^2}}{16\,r\,f}\right)$$

```
> diff(%,f);
```

$$\frac{\dfrac{64\,f+1024\,f^3-512\,r^2\,f}{32\,\sqrt{1+32\,f^2+256\,f^4-256\,r^2\,f^2}\;r\,f}-\dfrac{\sqrt{1+32\,f^2+256\,f^4-256\,r^2\,f^2}}{16\,r\,f^2}}{1+\dfrac{1+32\,f^2+256\,f^4-256\,r^2\,f^2}{256\,r^2\,f^2}}$$

```
> simplify(%);
```

$$\frac{16\,r\,(16\,f^2-1)}{(1+16\,f^2)\,\sqrt{1+32\,f^2+256\,f^4-256\,r^2\,f^2}}$$

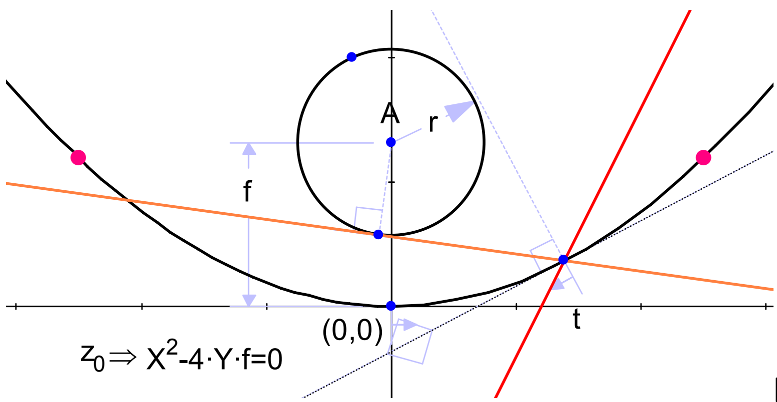
```
> solve(%,f);
```

$$\frac{1}{4}, \frac{-1}{4}$$

From the calculations, the optimal focal height is always when the focal height (*f)* is a fourth of the length of the diameter, regardless of the target size. This is also when the focal height is level with the boundaries of the solar cooker.

Problem: Spill Rates

Previously, we investigated the critical angle that reflects all rays within a diameter ratio of 1 to a pot with radius *r* and focal height *f.* After determining the boundary that light starts spilling over, it led to the question of how quickly the rest of the light misses the pot as the incoming angle decreases.

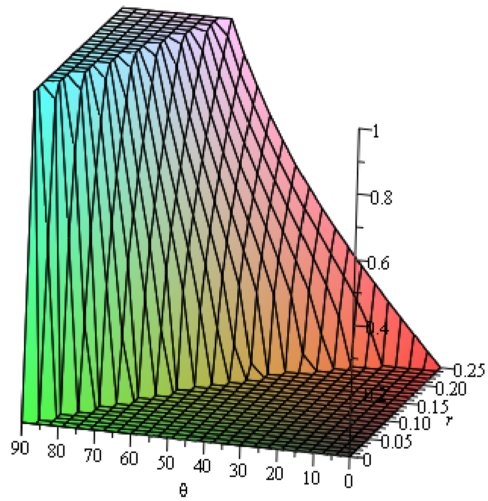


[8]

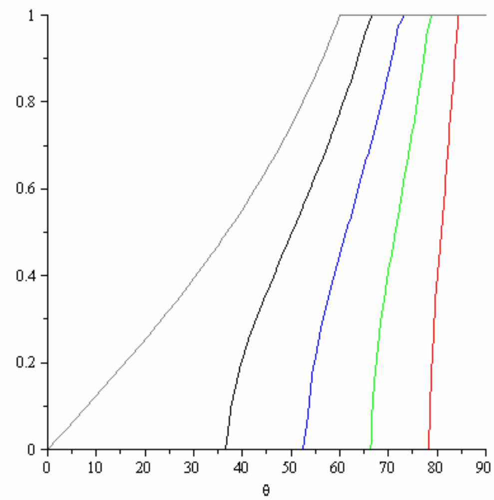
Using the above diagram [8] in conjunction with Maple, the percentage of light in the solar cooker with diameter ratio of 1 that hits the pot for a given pot radius and incoming angle was expressed in the piecewise function [9] below. Results are presented with the optical focal height.

$$spill := \begin{cases} 0 \\ \sqrt{-1+4\sqrt{r^2+\tan\left(\frac{1}{180}\theta\,\pi\right)^2 r^2}} \qquad \sqrt{-1+4\sqrt{r}} \\ 1 \end{cases}$$ [9]

The formula is graphed in 3D below [10]. The diagram [11] at the right is cross sections of the 3d graph at specific values of *r* (gray: 0.25, green: 0.20, blue: 0.15, black: 0.10, gray: 0.05).



[10]



[11]

When the radius of the pot is smaller, the percent efficiency decreases at a faster rate. When the radius is larger, the percent efficiency decreases at a slower rate. Now that we know how quickly light spills out of the pot in a parabolic solar cooker, the next step is to find a curve that may not be at perfect as the parabola as incoming angles around 90 degrees, but can hold more light in the pot at smaller incoming angle than the parabola because light spills out relatively quickly for a parabolic solar cooker.

Conclusion:

The solar concentrator is always most angle tolerant when the focal height is one-fourth of the the aperture length regardless of the target size. When the solar cooker is positioned this way, it will be able to concentrate all light to the target at the smallest incoming or widest angle possible for a given diameter and target. There isn't, however, an optimal target size. The larger the target, the more angle tolerant the solar concentrator is. However, the larger the target, the smaller the concentration ratio is. The trade-off between angle tolerance and solar concentration depends on the target size. To improve one, the other needs to be sacrificed. It is impossible to maximize both.

The results from graphing the efficiency of the concentrator at angles beyond its critical angle shows how the efficiency decreases very drastically with smaller targets, while larger targets are efficient at smaller incoming angles. The graph's applicability will be vital when comparing the parabola with other curves. Although there may not be another curve as perfect as the parabola when perfectly aligned, there may be curves that will have better efficiency at smaller incoming and wider angles.

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